Solving an Equation by Isolating a Variable

In this section we will learn how to solve for an unknown value in an equation by isolating a variable.

Before you continue working through this lesson, be sure that you have mastered the skills in the **Solving Simple Equations with One Unknown** lesson.

In addition to those skills, we will use the **Reverse Order of Operations** to isolate an unknown value **x**.

Remember that the **Order of Operations** is:

1. Brackets
2. Exponents
3. Multiplication or Division
4. Addition or Subtraction

If we were asked to complete the following calculation

$5 + (2)(3) + 4^2$

we would use the order of operations like this:

$5 + (2)(3) + \mathbf{4^2}$	Exponent first
$5 + (2)(3) + \mathbf{16}$	
$5 + \mathbf{(2)(3)} + 16$	then Multiplication
$5 + \mathbf{6} + 16$	
$\mathbf{5 + 6} + 16$	Finally, Addition
$\mathbf{11} + 16$	
$\mathbf{11 + 16}$	
$\mathbf{27}$	

If we want to isolate a variable **x** in an equation, we do the **Reverse Order of Operations.**

For example, let's isolate the variable **x** in the following equation to solve for **x**.

$10x - 2 = 38$

In order to isolate **x**, we must do the "Reverse Order of Operations" on the left side of the equation.

This means that in this case we will eliminate the **-2** first. We will do this by **adding 2** to both sides of the equation.

$10x - 2 \mathbf{+ 2} = 38 \mathbf{+ 2}$

$10x - 0 = 40$

$10x = 40$

$(10)(x) = 40$

Now we will **divide** both sides of the equation by **10** in order to isolate **x** and solve.

$$\frac{(\cancel{10})(x)}{\cancel{10}} = \frac{40}{10}$$

$$(1)(x) = 4$$

$$x = 4$$

Notice how we "reversed" the order of operations to solve for **x**.

We **added 2** before we **divided by 10**.

Let's try a more challenging equation to solve.

Use the reverse order of operations to determine the value of x in the following equation.

$$\frac{x+3}{5}=2$$

First, we need to acknowledge that **x + 3** is being divided by 5 and that it needs to be in a bracket.

$$\frac{(x+3)}{5}=2$$

Since the x + 3 is in a bracket, the first operation to deal with in the reverse order of operations is the **divided by 5**.

This means we will multiply both sides by **5**.

$$\left(\frac{(x+3)}{5}\right)(5)=(2)(5)$$

$$\left(\frac{(x+3)}{5}\right)\left(\frac{5}{1}\right)=10$$

$$\frac{(\cancel{5})(x+3)}{(\cancel{5})(1)}=10$$

$$(1)(x+3)=10$$

$$x+3=10$$

Once the **5** is dealt with, brackets around the **x + 3** can be removed.

Finally, to isolate the **x**, we will **subtract 3** from both sides of the equation.

$$x+3=10$$

$$x+3-3=10-3$$

$$x+0=7$$

$$x=7$$

Note that if we want to verify our answer of **x = 7**, we will use the **Order of Operations** on the left side of the equation.

For example,

$$\frac{x+3}{5}=2,\quad x=7$$

$$\frac{7+3}{5}=2$$

$$\frac{(7+3)}{5}=2$$

$$\frac{10}{5}=2$$

$$2=2$$

We have confirmed that **x = 7**

We are ready to do more challenging examples.

Example

1. Determine the value of **x** in the following equation.

$3x + 2 = 14$

Step 1: To isolate the **x**, first subtract **2** from both sides of the equation.

$3x + 2 \textbf{ - 2} = 14 \textbf{ - 2}$

$3x + 0 = 12$

$3x = 12$

Step 2: Now we can divide both sides of the equation by **3**.

$$\frac{3x}{3} = \frac{12}{3}$$

$$\frac{(\cancel{3})(x)}{\cancel{3}} = 4$$

$$(1)(x) = 4$$

$$x = 4$$

2. Determine the value of **x** in the following equation.

$7x - 4 = -39$

Step 1: To isolate the **x**, first add **4** to both sides of the equation.

$7x - 4 \textbf{ + 4} = -39 \textbf{ + 4}$

$7x - 0 = -35$

$7x = -35$

Step 2: Now we can divide both sides of the equation by **7**.

$$\frac{7x}{7} = \frac{-35}{7}$$

$$\frac{(\cancel{7})(x)}{\cancel{7}} = -5$$

$$(1)(x) = -5$$

$$x = -5$$

3. Determine the value of **x** in the following equation.

$15 - 3x = 27$

Step 1: To isolate the **x**, first subtract **15** from both sides of the equation.

$15 \textbf{ - 15} - 3x = 27 \textbf{ - 15}$

$0 - 3x = 12$

$-3x = 12$

Step 2: Now we can divide both sides of the equation by **-3**.

$$\frac{-3x}{-3} = \frac{12}{-3}$$

$$\frac{(\cancel{-3})(x)}{\cancel{-3}} = -4$$

$$(1)(x) = -4$$

$$x = -4$$

4. Determine the value of **x** in the following equation.

$$\frac{x}{4} + 7 = 12$$

Step 1: To isolate the **x**, first subtract **7** from both sides of the equation.

$$\frac{x}{4} + 7 - 7 = 12 - 7$$

$$\frac{x}{4} + 0 = 5$$

$$\frac{x}{4} = 5$$

Step 2: Now we can multiply both sides of the equation by **4**.

$$\left(\frac{x}{4}\right)(4) = (5)(4)$$

$$\left(\frac{x}{4}\right)\left(\frac{4}{1}\right) = 20$$

$$\frac{(\cancel{4})(x)}{(\cancel{4})(1)} = 20$$

$$\frac{(1)(x)}{1} = 20$$

$$x = 20$$

5. Determine the value of **x** in the following equation.

$$\frac{x-3}{5} = 4$$

Step 1: Since the whole numerator (**x – 3**) is being divided by **5**, we need to put a bracket around **x – 3**.

$$\frac{(x-3)}{5} = 4$$

Step 2: Next, we will multiply both sides of the equation by **5**.

$$\left(\frac{(x-3)}{5}\right)(5) = (4)(5)$$

$$\left(\frac{(x-3)}{5}\right)\left(\frac{5}{1}\right) = 20$$

$$\frac{(\cancel{5})(x-3)}{\cancel{5}} = 20$$

$$(1)(x-3) = 20$$

$$x - 3 = 20$$

Step 3: Finally, we will add **3** to both sides of the equation.

x - 3 **+ 3** = 20 **+ 3**

x - 0 = 23

x = 23